

The Empirical Beta Copula

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Abstract

Given a sample from a multivariate distribution F , the uniform random variates generated independently and rearranged in the order specified by the vector of ranks look like a sample from the copula of F . This idea can be regarded as Baker (2008)'s construction of copulas based on order statistics with the ranks being coefficients, and led us to define the empirical beta copula. It is then fairly easy to show that the empirical beta copula is a particular case of the empirical Bernstein copula (taking all the orders of the Bernstein polynomials equal to the sample size). The advantage is that we do not need any smoothing parameter. Also it is extremely simple to simulate a sample from the empirical beta copula.

We show that the empirical Bernstein copula is a genuine copula by providing (necessary and) sufficient conditions for a Bernstein transformation to be a copula. Furthermore, we establish the assumptions under which the standard asymptotic results hold for the empirical Bernstein copula. They are significantly weaker than those given in Janssen et al. (2012). Our Monte Carlo simulation study shows that there is an advantage of smoothing to improve finite-samples performance. It is found that in all cases, the empirical beta copula outperforms the empirical copula in terms of the bias and the integrated mean squared error. Compared with the empirical Bernstein copula with optimal smoothing rate, its performance is still significantly better in several cases, especially in terms of bias.