Prediction of the FIFA World Cup 2014 by a regularized random effects model

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Who will celebrate?

Who will cry?

Motivation

Prediction of the FIFA World Cup 2014 based on the match outcomes of preceding world cups (2002-2010).

Simplest approach: (binary) Bradley-Terry-Luce model

$$P(Y_{ij} = 1) = \frac{\exp(\alpha_i - \alpha_j)}{1 + \exp(\alpha_i - \alpha_j)}$$

⇒ logistic regression model for the probability that team i beats team j
⇒ α_i represents the strength of team i

Possible extensions/modifications:

- ordered response (including draws)
- Poisson response (number of goals)
- incorporation of covariates

Modified BTL-model

Extensions:

- team-specific random effects $b_i \sim N(0, \sigma_b^2)$ instead of fixed effects α_i
- Poisson response: number of goals scored in single matches of a team
- incorporation of several covariates

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$\Rightarrow$ **Model equation**:

$$
\begin{aligned}
y_{ijk} | \mathbf{x}_{ik}, \mathbf{x}_{jk}, b_i, b_j &\sim Pois(\lambda_{ijk}) \\
log(\lambda_{ijk}) &= \mathbf{x}_{ik}^T \beta_{team} + \mathbf{x}_{jk}^T \beta_{oppo} + b_i - b_j \, .
\end{aligned}
$$

- team $i, i = 1, \ldots, n$, n number of teams;
- opponent $j, j = 1, \ldots, n$;
- tournament $k, k = 1, 2, 3$;
- $\mathbf{x}_{ik}, \mathbf{x}_{jk}$ covariates of teams i and j, varying over tournaments
- y_{ijk} number of goals scored by team i against team j in tournament k

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  GDP per capita, population

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All variables are incorporated of both the team whose goals are considered and its opponent!

Data: FIFA World Cups 2002 - 2010

Structure of the data set

goals	team	opponent	age	rank	odds	...	age.o	rank.o	odds.o	...
0	France	Senegal	28.30	1	0.15	...	24.30	42	0.01	...
1	Senegal	France	24.30	42	0.01	...	28.30	1	0.15	...
1	Uruguay	Denmark	25.30	24	0.01	...	27.40	20	0.01	...
2	Denmark	Uruguay	27.40	20	0.01	...	25.30	24	0.01	...
1	Denmark	Senegal	27.40	20	0.01	...	24.30	42	0.01	...
1	Senegal	Denmark	24.30	42	0.01	...	27.40	20	0.01	...
0	France	Uruguay	28.30	1	0.15	...	25.30	24	0.01	...
0	Uruguay	France	25.30	24	0.01	...	28.30	1	0.15	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱

Structure of the design matrix of the random effects:

France	Senegal	Uruguay	Denmark	Paraguay	...
1	-1	0	0	0	...
-1	1	0	0	0	...
0	0	1	-1	0	...
0	0	-1	1	0	...
0	-1	0	1	0	...
0	1	0	-1	0	...
1	0	-1	0	0	...
-1	0	1	0	0	...
⋮	⋮	⋮	⋮	⋮	⋱

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⇒ to obtain nonetheless reasonable random effects estimates for both teams for the prediction of the FIFA World Cup 2014, we collect all teams that have only participated once in the tournaments between 2002 and 2014 in a group called **NEWCOMERS**.

This concerns the following 12 teams:

Angola, China, Czech Republic, Ireland, New Zealand, North Korea, Senegal, Slovakia, Togo, Trinidad & Tobago, Turkey, Ukraine

The **generalized linear mixed model** (GLMM) has the form

$$g(\mu_{it}) = \boldsymbol{x}_{it}^T \beta + \boldsymbol{z}_{it}^T \boldsymbol{b} = \eta_{it},$$

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Notations:

- y_{it} observation t in cluster i, $i = 1, \ldots, n$, $t = 1, \ldots, T_i$.
- $\boldsymbol{x}_{it}$ covariate vector associated with fixed effects $\beta_0, \beta_1, \ldots, \beta_p$.
- $\boldsymbol{z}_{it}^T$ covariate vector associated with random effects.
- $y_{i1}, \ldots, y_{iT_i}$ are conditionally independent, with means $\mu_{it} = E(y_{it} | \boldsymbol{x}_{it}, \boldsymbol{z}_{it}, \mathbf{b})$ and variances $var(y_{it} | \mu_{it}) = \phi \upsilon(\mu_{it})$, $\upsilon(.)$ is a known variance function and ϕ is a scale parameter.
- $\mathbf{b}^T = (\mathbf{b}_1^T, \ldots, \mathbf{b}_n^T)$ cluster-specific random effects of dimension s, $\boldsymbol{b}_i \sim N(\mathbf{0}, \boldsymbol{Q}(\boldsymbol{\varrho}))$ with covariance matrix $\boldsymbol{Q}(\boldsymbol{\varrho})$.

Likelihood

Exponential Family

The conditional density of y_{it}, given the explanatory variables $\boldsymbol{x}_{it}$ and $\boldsymbol{z}_{it}$ and the random effects $\boldsymbol{b}$, is assumed to be of **exponential family type**

$$f(y_{it}|\boldsymbol{x}_{it}, \boldsymbol{z}_{it}, \boldsymbol{b}) = \exp\left\{ \frac{(y_{it}^T \theta_{it} - \kappa(\theta_{it}))}{\phi} + c(y_{it}, \phi) \right\},$$

with natural parameter $\theta_{it} = \theta(\mu_{it})$, log normalization constant $c(.)$ and dispersion parameter ϕ.

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Define the parameter vector $\boldsymbol{\delta}^T = (\beta^T, \mathbf{b}^T)$.

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The parameter vector of the covariance structure $\boldsymbol{\varrho}$ together with the dispersion parameter ϕ are collected in $\boldsymbol{\gamma}^T = (\phi, \boldsymbol{\varrho}^T)$.

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For known $\boldsymbol{\gamma}$ and given data $\mathbf{y}_1^T, \ldots, \mathbf{y}_n^T$, estimation of $\boldsymbol{\delta}$ is based on maximizing the corresponding log-likelihood.

Log-Likelihood

We consider the following log-likelihood

$$l(\boldsymbol{\delta}, \boldsymbol{\gamma}) = \log\left(\int f(\mathbf{y}|\boldsymbol{\delta}, \boldsymbol{\gamma}) p(\mathbf{b}, \boldsymbol{\gamma}) d\mathbf{b}\right),$$

where $p(\mathbf{b}, \boldsymbol{\gamma})$ denotes the density of the random effects.

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Laplace approximation (Breslow & Clayton, 1993) yields the log-likelihood

$$l^{\text{app}}(\boldsymbol{\delta},\boldsymbol{\gamma}) = \sum_{i=1}^{n}\log f(\mathbf{y}_i|\boldsymbol{\delta},\boldsymbol{\gamma}) - \frac{1}{2}\mathbf{b}^T\mathbf{Q}_{\mathbf{b}}(\varrho)^{-1}\mathbf{b}\,,$$

with $\mathbf{Q}_{\mathbf{b}} = diag(\mathbf{Q},\ldots,\mathbf{Q})$.

We obtain estimates of the parameter vector $\boldsymbol{\delta}$ via Fisher scoring:

$$\hat{\boldsymbol{\delta}}^{(l)} = \hat{\boldsymbol{\delta}}^{(l-1)} + (\mathbf{F}^{\text{app}}(\hat{\boldsymbol{\delta}}^{(l-1)}))^{-1}\mathbf{s}^{\text{app}}(\hat{\boldsymbol{\delta}}^{(l-1)}).$$

L_1-penalized GLMMs

The log-likelihood is expanded to include the penalty term $\lambda \sum_{i=1}^{p} |\beta_i|$:

$$l^{\text{pen}}(\boldsymbol{\delta}, \boldsymbol{\gamma}) = l^{\text{app}}(\boldsymbol{\delta}, \boldsymbol{\gamma}) - \lambda \sum_{i=1}^{p} |\beta_i|. \tag{1}$$

For given $\hat{\gamma}$ the optimization problem reduces to

$$\hat{\boldsymbol{\delta}} = \underset{\boldsymbol{\delta}}{\text{argmax}}\ l^{\text{pen}}(\boldsymbol{\delta}, \hat{\gamma}) = \underset{\boldsymbol{\delta}}{\text{argmax}}\ \left[l^{\text{app}}(\boldsymbol{\delta}, \hat{\gamma}) - \lambda \sum_{i=1}^{p} |\beta_i| \right]. \tag{2}$$

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Incorporation of categorical predictors: $\sum_{g=1}^{G} \lambda_g ||\boldsymbol{\beta}_{\boldsymbol{I_g}}||_2$ with $\lambda_g = \lambda\sqrt{\text{df}_g}$

1. *Initialization*
   Compute starting values $\hat{\beta}^{(0)}, \hat{\mathbf{b}}^{(0)}, \hat{\gamma}^{(0)}$ and set $\hat{\boldsymbol{\eta}}^{(0)} = \mathbf{X}\hat{\beta}^{(0)} + \mathbf{Z}\hat{\mathbf{b}}^{(0)}$.

2. *Iteration*
   For $l = 1, 2, \ldots$ until convergence:

   a) *Calculation of the log-likelihood gradient for given $\hat{\gamma}^{(l-1)}$*

      With $\mathbf{s}(\boldsymbol{\delta}) = \partial l^{\mathbf{app}}(\boldsymbol{\delta})/\partial\boldsymbol{\delta}$ derive:

$$s^{\mathbf{pen}}_{\beta_{\mathbf{0}}}(\hat{\boldsymbol{\delta}}^{(l-1)}) = s_{\beta_{\mathbf{0}}}(\hat{\boldsymbol{\delta}}^{(l-1)}), \qquad s^{\mathbf{pen}}_{\mathbf{b}_i}(\hat{\boldsymbol{\delta}}^{(l-1)}) = s_{\mathbf{b}_i}(\hat{\boldsymbol{\delta}}^{(l-1)}).$$

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   For $\beta_i, i = 1, \ldots, p$ derive:

$$s^{\mathbf{pen}}_{\beta_i}(\hat{\boldsymbol{\delta}}^{(l-1)}) = \begin{cases} s_{\beta_i}(\hat{\boldsymbol{\delta}}^{(l-1)}) - \lambda \,\text{sign}\,(\hat{\beta}_i^{(l-1)}) & \text{if } \hat{\beta}_i^{(l-1)} \neq 0 \\ s_{\beta_i}(\hat{\boldsymbol{\delta}}^{(l-1)}) - \lambda \,\text{sign}\,(s_{\beta_i}(\hat{\boldsymbol{\delta}}^{(l-1)})) & \text{if } \hat{\beta}_i^{(l-1)} = 0, |s_{\beta_i}(\hat{\boldsymbol{\delta}}^{(l-1)})| > \lambda \\ 0 & \text{otherwise.} \end{cases}$$

b) *Calculation of the directional second derivative*
The directional second derivative is given for every $\boldsymbol{\delta}$ and every direction vector $\mathbf{v} \in \mathbb{R}^{p+1+ns}$ by

$$l''_{\mathbf{pen}}(\boldsymbol{\delta}; \mathbf{v}) = \mathbf{v}^T \mathbf{F}^{\mathbf{pen}}(\boldsymbol{\delta})\mathbf{v},$$

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with $\mathbf{F}^{\mathbf{pen}}(\boldsymbol{\delta}) = \mathbf{A}^T \mathbf{W}(\boldsymbol{\delta})\mathbf{A} + \mathbf{K}$ and $\mathbf{A} := [\mathbf{X}, \mathbf{Z}]$, $\mathbf{W}(\boldsymbol{\delta}) = \mathbf{D}(\boldsymbol{\delta})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\delta})\mathbf{D}(\boldsymbol{\delta})^T$, $\mathbf{K} = diag(0, \ldots, 0, \mathbf{Q}^{-1}, \ldots, \mathbf{Q}^{-1})$, $\mathbf{D}(\boldsymbol{\delta}) = \partial h(\boldsymbol{\eta})/\partial\boldsymbol{\eta}$ and $\boldsymbol{\Sigma}(\boldsymbol{\delta}) = cov(\mathbf{y}|\boldsymbol{\delta})$.

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c) *Optimum of directional 2. order Taylor approximation*
Approximate l_{pen} locally from $\boldsymbol{\delta}$ in direction of the gradient:

$$l_{\text{pen}}(\boldsymbol{\delta} + t s^{\text{pen}}(\boldsymbol{\delta})) \approx l_{\text{pen}}(\boldsymbol{\delta}) + t l'_{\text{pen}}(\boldsymbol{\delta}, s^{\text{pen}}(\boldsymbol{\delta})) + \frac{1}{2} t^2 l''_{\text{pen}}(\boldsymbol{\delta}, s^{\text{pen}}(\boldsymbol{\delta}))$$

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$$t_{\mathbf{opt}}^{(l-1)} = \frac{||\mathbf{s}^{\mathbf{pen}}(\hat{\boldsymbol{\delta}}^{(l-1)})||_2}{l''_{\mathbf{pen}}(\hat{\boldsymbol{\delta}}^{(l-1)}, \mathbf{s}^{\mathbf{pen}}(\hat{\boldsymbol{\delta}}^{(l-1)}))},$$

$||\cdot||_2$ denoting the L_2-norm.

$$t_{\textbf{edge}}^{(l-1)} = \min_i \left\{ -\frac{\hat{\delta}_i^{(l-1)}}{s_i^{\textbf{pen}}(\hat{\boldsymbol{\delta}}^{(l-1)})} : \text{sign}(\hat{\delta}_i^{(l-1)}) = -\text{sign}[s_i^{\textbf{pen}}(\hat{\boldsymbol{\delta}}^{(l-1)})] \neq 0 \right\}$$

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d) *Update*

$$\hat{\boldsymbol{\delta}}^{(l)} = \begin{cases} \hat{\boldsymbol{\delta}}^{(l-1)} + t_{\text{edge}}^{(l-1)} \mathbf{s}^{\text{pen}}(\hat{\boldsymbol{\delta}}^{(l-1)}) & \text{if } t_{\text{opt}}^{(l-1)} \geq t_{\text{edge}}^{(l-1)} \\ \hat{\boldsymbol{\delta}}^{(l-1)} + t_{\text{opt}}^{(l-1)} \mathbf{s}^{\text{pen}}(\hat{\boldsymbol{\delta}}^{(l-1)}) & \text{otherwise.} \end{cases}$$

e) *Computation of variance-covariance components*

- Approximate **EM**-type estimate:

$$\hat{\mathbf{Q}}^{(l)} = \frac{1}{n}\sum_{i=1}^{n}(\hat{\mathbf{V}}_{ii}^{(l)} + \hat{\mathbf{b}}_i^{(l)}(\hat{\mathbf{b}}_i^{(l)})^T), \qquad \text{with} \qquad \hat{\mathbf{V}}_{ii}^{(l)} \in (\mathbf{F}^{\mathbf{pen}}(\hat{\boldsymbol{\delta}}^{(l)}))^{-1}.$$

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- Approximate **REML**-type estimate: maximize

$$\begin{aligned} l(\mathbf{Q_b}) \quad = \quad & -\frac{1}{2}\log(|\mathbf{V}(\hat{\boldsymbol{\delta}}^{(l)})|) - \frac{1}{2}\log(|\mathbf{X}^T\mathbf{V}^{-1}(\hat{\boldsymbol{\delta}}^{(l)})\mathbf{X}|) \\ & -\frac{1}{2}(\tilde{\boldsymbol{\eta}}(\hat{\boldsymbol{\delta}}^{(l)}) - \mathbf{X}\hat{\boldsymbol{\beta}}^{(l)})^T\mathbf{V}^{-1}(\hat{\boldsymbol{\delta}}^{(l)})(\tilde{\boldsymbol{\eta}}(\hat{\boldsymbol{\delta}}^{(l)}) - \mathbf{X}\hat{\boldsymbol{\beta}}^{(l)}) \end{aligned}$$

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with respect to $\mathbf{Q_b}$.

$$\begin{aligned} &\mathbf{V}(\boldsymbol{\delta}) = \mathbf{W}^{-1}(\boldsymbol{\delta}) + \mathbf{Z}\mathbf{Q_b}\mathbf{Z}^T, && \mathbf{Q_b} = diag(\mathbf{Q}, \dots, \mathbf{Q}), \\ &\tilde{\boldsymbol{\eta}}(\boldsymbol{\delta}) = [\mathbf{X}, \mathbf{Z}]\,\boldsymbol{\delta} + \mathbf{D}^{-1}(\boldsymbol{\delta})(\mathbf{y} - \boldsymbol{\mu}(\boldsymbol{\delta})), && \mathbf{D}(\boldsymbol{\delta}) = \frac{\partial h(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}}, \\ &\mathbf{W}(\boldsymbol{\delta}) = \mathbf{D}(\boldsymbol{\delta})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\delta})\mathbf{D}(\boldsymbol{\delta})^T, && \boldsymbol{\Sigma}(\boldsymbol{\delta}) = \text{cov}(\mathbf{y}|\boldsymbol{\delta}). \end{aligned}$$

The overdispersion parameter Φ is estimated by use of Pearson residuals $\hat{r}_{it} = (y_{it} - \hat{\mu}_{it})/(v(\hat{\mu}_{it}))^{\frac{1}{2}}$ as

$$\hat{\Phi} = \frac{1}{N - \mathrm{df}} \sum_{i=1}^{n} \sum_{t=1}^{T_i} \hat{r}_{it}^2, \qquad N = \sum_{i=1}^{n} T_i,$$

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Call in R using the package glmmLasso:

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>lasso.obj <- glmmLasso(goals ~ rank + rank.o + continent + continent.o ...,
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$$\hat{\Phi} = \frac{1}{N - \text{df}} \sum_{i=1}^{n} \sum_{t=1}^{T_i} \hat{r}_{it}^2, \qquad N = \sum_{i=1}^{n} T_i,$$

where the degrees of freedom (df) correspond to the trace of the hat-matrix.

Call in R using the package glmmLasso:

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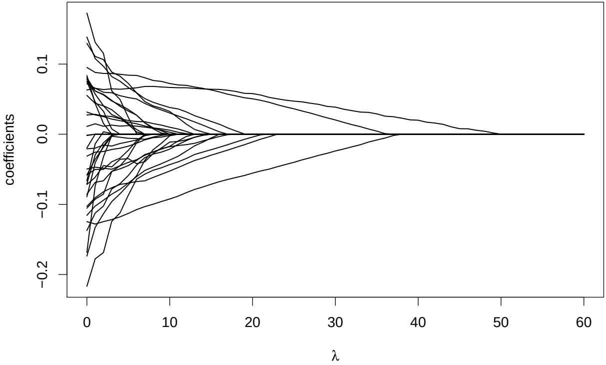
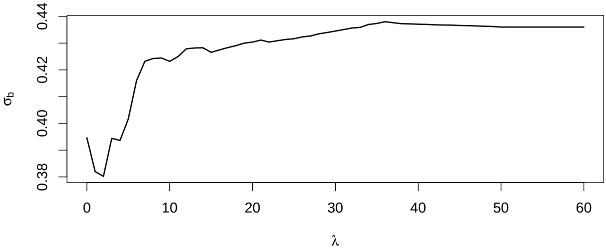
Coefficient paths for the FIFA World Cup data





Selection of tuning parameter

Idea: **leave-one-out cross validation**, especially designed for soccer data

- for the prediction of single matches we are mainly interested in the general match outcome (win, draw, defeat)
- single matches are treated as observations
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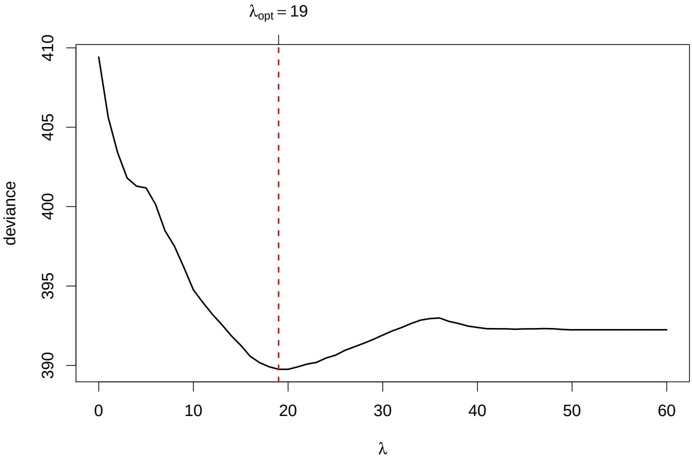
Let the omitted observation/match be a match of team k against team l:

- predictions $\hat{\mu}_k$ and $\hat{\mu}_l$ for a given model
  $\Rightarrow \hat{p}_1 = P(Y_k > Y_l), \hat{p}_2 = P(Y_k = Y_l)$ and $\hat{p}_3 = P(Y_k < Y_l)$
- for a given match outcome $\omega \in \{1, 2, 3\}$, the quantity

$$\hat{p}_1^{\delta_{1\omega}} \hat{p}_2^{\delta_{2\omega}} \hat{p}_3^{\delta_{3\omega}}$$

  represents the likelihood score of the corresponding match
- $\Rightarrow$ deviance for multinomial logit model

Leave-one-out cross validation



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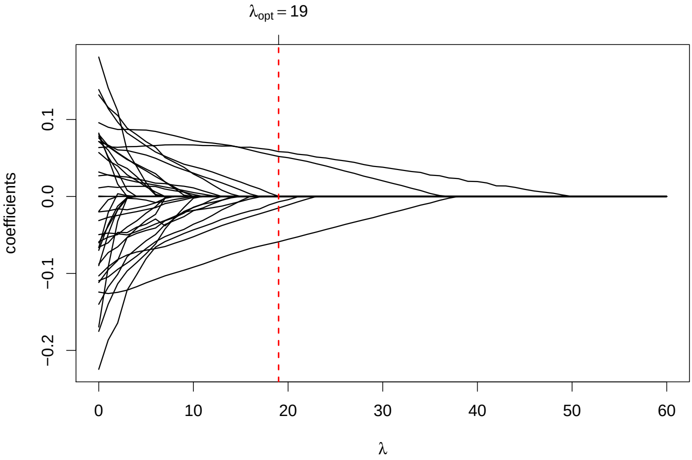
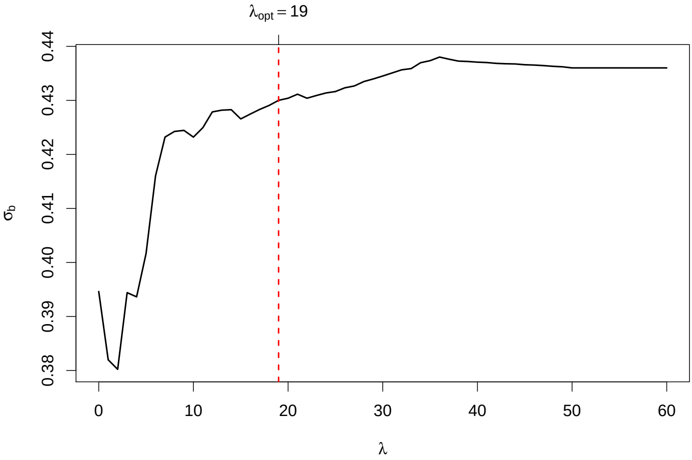


Standard deviation of the random effects



Final model based on LOOCV

Team covariates

- **Economic Factors:**
  GDP per capita, population
- **Sportive Factors:**
  Fairness, ODDSET odds, FIFA rank, confederation
- **Home advantage:**
  host of the world cup, same continent as host
- **Factors describing the team's structure**
  Maximum number of teammates, Second maximum number of teammates, average age, number of CL players, number of Europa League players, number of players abroad
- **Factors describing the team's coach**
  age, nationality, tenure

Final model based on LOOCV

Opponent covariates

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Estimation results

(Intercept)	Nation.Coach	Population	Rank.oppo
0.0169	0.0004	-0.0151	0.0521

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1.	BRA	0.74	11.	URU	0.24	21.	RSA	0.00	31.	POL	-0.28
2.	GER	0.72	12.	SWE	0.15	22.	CRO	0.00	32.	SVN	-0.31
3.	ARG	0.56	13.	MEX	0.15	23.	AUS	-0.01	33.	SRB	-0.33
4.	NED	0.44	14.	FRA	0.14	24.	JPN	-0.05	34.	RUS	-0.35
5.	POR	0.44	15.	PAR	0.12	25.	NGA	-0.06	35.	CRC	-0.35
6.	ESP	0.37	16.	ECU	0.10	26.	DEN	-0.12	36.	CMR	-0.36
7.	ITA	0.30	17.	KOR	0.10	27.	NEW	-0.13	37.	ALG	-0.37
8.	ENG	0.29	18.	GHA	0.09	28.	HON	-0.19	38.	IRN	-0.41
9.	SUI	0.27	19.	CHI	0.06	29.	BEL	-0.20	39.	TUN	-0.73
10.	CIV	0.25	20.	USA	0.05	30.	GRE	-0.27	40.	KSA	-1.09

Goodness-of-fit (in sample)

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Corresponding **deviance scores** for `glmmLasso`, `grplasso` and ODDSET odds:

glmmLasso		grplasso	ODDSET odds
AIC	LOOCV		
334.3549	333.1851	336.7894	365.4724

Simulation of the tournament progress

- every single match outcome can be simulated by drawing from the respective Poisson distributions
- for every group, the exact standing after the group stage can be calculated

⇒ decision on qualification for round of 16 according to FIFA rules

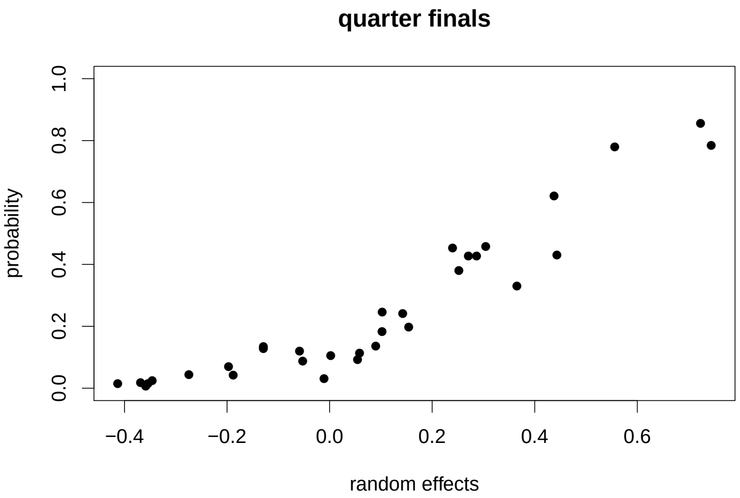
- draws in knockout stage → team with higher probability wins

⇒ 10000 simulation runs for FIFA World Cup 2014

Probabilities for FIFA World Cup Winner 2014

team		$\hat{p}_{\text{glmmLasso}}$	$\hat{p}_{\text{Oddset}}$	team		$\hat{p}_{\text{glmmLasso}}$	$\hat{p}_{\text{Oddset}}$
1.	BRA	0.3000	0.2028	17.	KOR	0.0015	0.0024
2.	GER	0.2778	0.1420	18.	COL	0.0007	0.0394
3.	ARG	0.1699	0.1420	19.	CRO	0.0007	0.0071
4.	POR	0.0696	0.0237	20.	USA	0.0007	0.0071
5.	NED	0.0469	0.0355	21.	BEL	0.0003	0.0592
6.	ESP	0.0285	0.1092	22.	JPN	0.0003	0.0047
7.	URU	0.0210	0.0284	23.	BIH	0.0002	0.0047
8.	ITA	0.0202	0.0355	24.	AUS	0.0002	0.0024
9.	ENG	0.0173	0.0355	25.	HON	0.0002	0.0005
10.	SUI	0.0171	0.0071	26.	RUS	0	0.0118
11.	CIV	0.0102	0.0071	27.	GRE	0	0.0071
12.	FRA	0.0045	0.0355	28.	ALG	0	0.0071
13.	ECU	0.0042	0.0071	29.	CRC	0	0.0071
14.	CHI	0.0033	0.0203	30.	NGA	0	0.0035
15.	MEX	0.0029	0.0071	31.	CMR	0	0.0024
16.	GHA	0.0018	0.0071	32.	IRN	0	0.0005

Influence of random effects on tournament success



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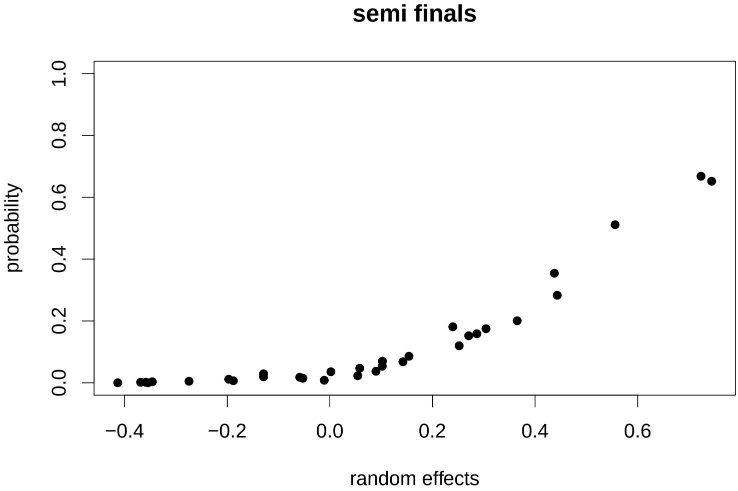

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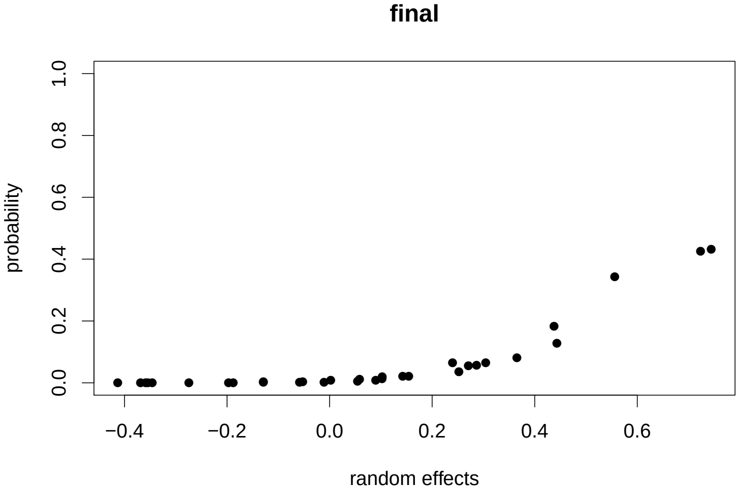

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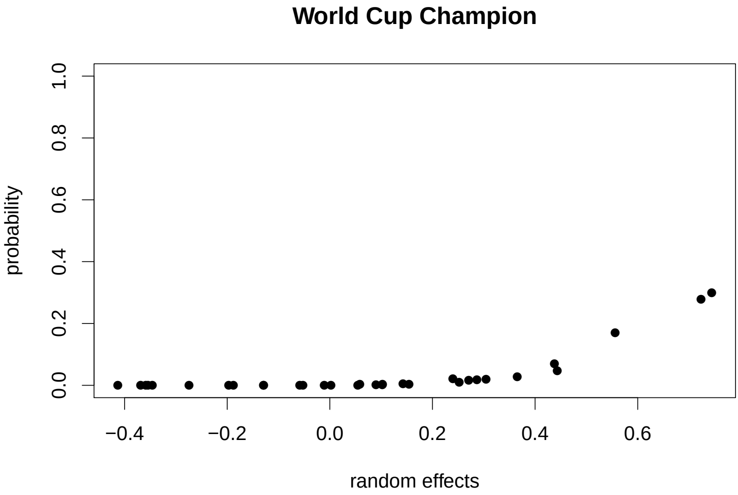
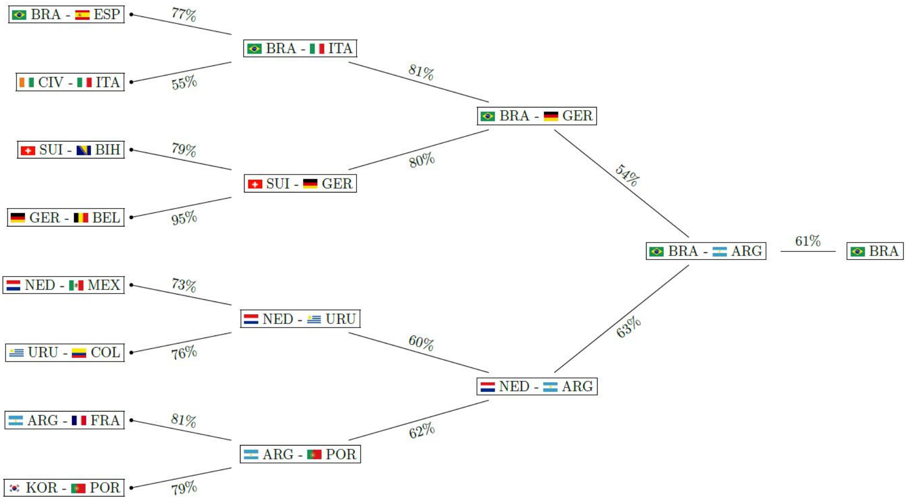


Most probable tournament outcome: group standings

Group A 46%	Group B 25%	Group C 24%	Group D 17%	Group E 19%	Group F 38%	Group G 42%	Group H 27%
1. BRA	1. NED	1. CIV	1. URU	1. SUI	1. ARG	1. GER	1. KOR
2. MEX	2. ESP	2. COL	2. ITA	2. FRA	2. BIH	2. POR	2. BEL
CRO	CHI	JPN	ENG	ECU	NGA	GHA	RUS
CMR	AUS	GRE	CRC	HON	IRN	USA	ALG

Most probable tournament outcome: knockout stage



Summary

Model:

- the `glmmLasso` algorithm allows to include complex random effects structures, e.g. for paired comparison data;
- simulations and applications show that the proposed procedure provides **convincing estimators**.

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FIFA World Cup application:

- huge impact of team-specific random effects $\Rightarrow$ most variables concerning teams strength are not selected;
- unexpected set of selected variables;
- three top favorites (not including Spain!).

References & Software

Goeman, J. J. (2012). *penalized.* R package version 0.9-42.

Goeman, J. J. (2010). L_1 Penalized Estimation in the Cox Proportional Hazards Model. *Biometrical Journal 52*, 70–84.

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Die Hoffnung stirbt zuletzt!